Homework 8: Sections 8.1 - 8.2
KEY

STA209-04: Applied Statistics

Assigned: 04/19/2019
Due: 04/26/2019

Total Possible Points: 43

From the Book:

Questions: 8.16, 8.25, 8.29 (a,b, and d), 8.57, 8.59, 8.61

8.16 [7 pts] Exercise 3.96 introduces a study designed to examine the effect of doing synchronized movements (such as marching in step or doing synchronized dance steps) and the effect of exertion on many different variables, including how close participants feel to others in their group. In the study, high school students in Brazil were randomly assigned to an exercise with either high synchronicity (HS) or low synchronicity (LS) and also either to high exertion (HE) or low exertion (LE). Thus, there are four groups: HS+HE, HS+LE, LS+HE, and LS+LE. Closeness is measured on a 7-point Likert scale (1 = least close to 7 = most close), and the response variable is the change in how close participants feel to those in their group using the rating after the exercise minus the rating before the exercise. The data are stored in SynchronizedMovement and output for an ANOVA test is shown below, along with some summary statistics.

Analysis of Variance

Source	DF	SS	MS	F-Value	P-Value
Group	3	27.04	9.012	2.77	0.042
Error	256	831.52	3.248		
Total	259	858.55			

Group	N	Mean	StDev
HS+HE	72	0.319	1.852
HS+LE	64	0.328	1.861
LS+HE	66	0.379	1.838
LS+LE	58	-0.431	1.623

a) [1 pts] In both groups with high synchronization (HS) does mean closeness rating go up or down after the synchronized exercise?

Since the change in closeness was measured by subtracting the first reading from the one obtained after the exercise, a positive mean would indicate an increase in closeness rating. Positive means are observed in both HS groups, meaning that closeness rating increased for these groups.

b) [2 pts] In the groups with low synchronization (LS), does mean closeness rating go up or gown if the group engages in high exertion (HE) exercise? How about if the group engages in low exertion (LE) exercise?

The mean closeness rating goes up if the LS group engages in HE exercise. The opposite is true if the group engages in LE exercise (i.e. the mean closeness rating goes down).

c) [1 pts] How many students were included in the analysis?

There were 260 students included in this analysis.

d) [2 pts] At a 5% level, what is the conclusion of the test? If there is a difference in means, indicate where the difference is likely to be.

At a 5% confidence level, we reject the null hypothesis of equal means across the four groups. We conclude that at least one group mean is different from the rest. Given that all means are positive and similar in magnitude except for LS+LE, it is likely that this group is where our difference lies.

e) [1 pts] At a 1% level, what is the conclusion of the test?

At a 1% level, we would fail to reject our null hypothesis. We would conclude that we had marginal, but not sufficient, evidence of a difference among the mean change in closeness rating among the four groups studied.

8.25 [10 pts] Exercises 8.25 to 8.27 describe three different experiments investigating the effect of drawing on memory. In each experiment participants were randomly divided into three groups and shown a list of 10 words to memorize. The response variable is the number of words participants are able to recall. In the first experiment, participants were instructed to either draw an image for each word, write a list of attributes for each word, or write the word. The summary statistics for the number of words recalled (out of 10) are shown in Table 8.10.

Table 8.10

Group	n	**Mean**	**St.Dev.**
Draw	16	4.8	1.3
List attributes	16	3.4	1.6
Write	16	3.2	1.2
Overall	48	3.8	1.527

Construct an ANOVA table to assess the difference in mean word recall between the three groups and answer the following questions. [5 pts]

We first construct an ANOVA table by using the information for Table 8.10:

Analysis of Variance

Source	DF	SS	MS	F-Value	P-Value
Group	2	$SST - SSE = 24.24$	$SSG/DF = 12.12$	$MSG/MSE = 6.38$	0.004
Error	45	$\sum_i (n_i - 1)s_i^2 = 85.35$	$SSG/DF = 1.90$		
Total	47	$(n-1)s^2 = 109.59$			

a) [1 pts] Which method had the highest mean recall in the sample? Which had the lowest?

The "Draw" group had the highest mean recall in the sample, and the "Write" group had the lowest.

b) [1 pts] What is the F-statistic?

The F-statistic is 6.38.

c) [1 pts] What is the p-value?

The p-value is 0.004.

d) [1 pts] At a 5% level, what is the conclusion of the test?

At the 5% level, we reject the null hypothesis of no mean difference. The mean word recall differs among the three methods.

e) [1 pts] According to this experiment, does it matter what you do if you want to memorize a list of words, and, if it matters, what should you do?

According to this experiment, the choice of method to memorize a list of words does matter. Given that "Draw" had the highest mean, it is recommended that you draw an image for each word if you want to memorize a list.

8.29 [7 pts] Studies have shown that exposure to light at night is harmful to human health. Data 4.1 on page 258 introduces a study in mice showing that dim light at night has an effect on weight gain after just three weeks. In the full study, mice were randomly assigned to live in one of three light conditions: LD had a standard light/dark cycle, LL had bright light all the time, and DM had dim light when there normally would have been darkness.

The mice in the study had body mass measured throughout the study. Computer output showing an analysis of variance table to test for a difference in mean body mass gain (in grams) after four weeks between mice in the three different light conditions is shown. We see in Exercise 8.28 that the conditions for ANOVA are met, and we also find the summary statistics fr each experimental group there.

One-way ANOVA:BM Gain versus Light

Source	DF	SS	MS	F	P
Light	2	113.08	56.54	8.38	0.002
Error	24	161.84	6.74		
Total	26	274.92			

a) [1 pts] State the null and alternative hypotheses.

$H_0 : \mu_{LD} = \mu_{LL} = \mu_{DM}$; H_A : At least one $\mu_i \neq \mu_j$

b) [3 pts] What is the F-statistic? What is the p-value? What is the conclusion of the test?

The F statistic is 8.38, and the p-vallue is 0.002. We reject the null hypothesis and conclude that the mean BM gain differs among the different light exposures.

d) [1 pts] Can we conclude that there is a cause-and-effect relationship between the variables? Why or why not?

Yes, these data were obtained from a randomized experiment.

8.57 [7 pts] Exercise 6.219 on page 465 introduces a study showing that exercise appears to offer some resiliency against stress, and Exercise 8.19 on page 556 follows up on this introduction. In the study, mice were randomly assigned to live in an enriched environment (EE), a standard environment (SE), or an impoverished environment (IE) for several weeks. Only the enriched environment provided opportunities for exercise. Half the mice then remained in their home cage (HC) as control groups while half were subjected to stress (SD). The researchers were interested in how resilient the mice were in recovering from the stress. One measure of mouse anxiety is the amount of time hiding in a dark compartment, with mice who are more anxious spending more time in darkness. The amount of time (in seconds) spend in darkness during one trial is recorded for all the mice and the means and the results of the ANOVA analysis are shown. There are eight mice in each of the six groups.

Group:	IE:HC	SE:HC	EE:HC	IE:SD	SE:SD	EE:SD
Mean:	192	196	205	392	438	231

Source	DF	SS	MS	F	P
Light	5	481776	96355.2	39.0	0.000
Error	42	177835	2469.9		
Total	47	659611			

a) [3 pts] Is there a difference between the groups in the amount of time spent in darkness? Between which two groups are we most likely to find a difference in mean time spent in darkness? Between which two groups are we least likely to find a difference?

Given the statistically significant F statistic shown in the ANOVA table, we would conclude that there is a difference between the groups in the amount of time spent in darkness. We are most likely to find a difference in mean time spent in darkness when comparing the SD:SD group to the IE:HC group. These have the highest and lowest means respectively. We are least likely to find a difference in means between IE:HC and SE:HC since their respective means are very similar to one another.

b) [2 pts] By looking at the six means, where do you think the differences are likely to lie?

Looking at the six means, the differences are likely to be found between the SD and HC groups. Across the HC groups, I do not anticipate differences to be found given the proximity each mean is to one another. This is in contrast to the SD groupings in which each is drastically different (392, 438, 231).

c) [2 pts] Test to see if there is a difference in mean time spent in darkness between the IE:HC group and the EE:SD group (that is, impoverished but not stressed vs enriched but stressed).

We use the ANOVA table and compute our test statistic:

$$t_{test} = \frac{\bar{x}_{IE:HC} - \bar{x}_{EE:SD}}{\sqrt{MSE\left(\frac{1}{n_{IE:HC}} + \frac{1}{n_{EE:SD}}\right)}} = -1.569$$

We use a $t_{df=42}$ distribution to obtain a p-value of 0.124. We fail to reject the null hypothesis of no difference in mean.

8.59 [2 pts] Exercise 8.16 on page 554 looks at possible differences in ratings of closeness to a group after doing a physical activity that involves either high or low levels of synchronization (HS or LS) and high or low levels of exertion (HE or LE). Students were randomly assigned to one of four groups with different combinations of these variables, and the change in their ratings of closeness to their group (on a 1 to 7 scale) were recorded. The data are stored in SynchronizedMovement and the means for each treatment group are given below, along with an ANOVA table that indicates a significant difference in the means at a 5% level.

Group	N	Mean	StDev
HS+HE	72	0.319	1.852
HS+LE	64	0.328	1.861
LS+HE	66	0.379	1.838
LS+LE	58	-0.431	1.623

Analysis of Variance

Source	DF	SS	MS	F-Value	P-Value
Group	3	27.04	9.012	2.77	0.042
Error	256	831.52	3.248		
Total	259	858.55			

The first three means look very similar, but the LS+LE group looks quite a bit different from the others. Is that a significant difference? Test this by comparing the mean difference in change in closeness ratings between the synchronized, high exertion activity group (HS+HE) and the non-synchronized, low exertion activity group (LS+LE).

We use the ANOVA table and compute our test statistic:

$$t_{test} = \frac{\bar{x}_{HS+HE} - \bar{x}_{LS+LE}}{\sqrt{MSE\left(\frac{1}{n_{HS+HE}} + \frac{1}{n_{LS+LE}}\right)}} = 2.358$$

We use a $t_{df=256}$ distribution to obtain a p-value of 0.0192. We reject the null hypothesis of no difference in means between the HS+HE and LS+LE groups.

8.61 [10 pts] Use Fisher's LSD, as described in Exercise 8.60, to discuss differences in mean time mice spend in darkness for the six combinations of environment and stress that produce the output in Exercise 8.57.

Description of Fisher's LSD from Exercise 8.60:

One way to "automate" pairwise comparisons that works particularly well when the sample sizes are balanced is to compute a single value that can serve as a threshold for when a pair of sample means are far enough apart to suggest that the population means differ between those two groups. One such value is called the Fisher's *Least Significant Difference* or *LSD* for short.

$$LSD = t_{crit}\sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

You may recognize this as the margin of error for a confidence interval for a difference in two means after doing an ANOVA. That is exactly how we compute it. Recall that the test for a pair of means will show a significant difference exactly when the confidence interval fails to include zero. The confidence level should be matched to the significance level of the test (for example, a 95% confidence interval corresponds to a 5% significance level). IF the difference in two group means (in absolute value) is smaller than the LSD margin of error, the confidence interval will have one positive and one negative endpoint. Otherwise, the interval will stay either all positive or all negative and we conclude the two means differ: Reject H_0 and conclude the two means differ $\iff |\bar{x}_i - \bar{x}_j| > LSD$.

We first compute the LSD:

$$LSD = t_{df=42,0.025}\sqrt{MSE(\frac{1}{n_i} + \frac{1}{n_j})} = 2.018\sqrt{2469.9 * \frac{2}{8}} = 50.14$$

Next, we compute all 15 pairwise (absolute) differences and compare to the LSD. A table showing these differences and whether each are statistically significant is provided below. Looking across

Groups	$\lvert\bar{x}_i - \bar{x}_j\rvert$	Significant at $\alpha = 0.05$?
IE:HC - SE:HC	$\lvert 192 - 196\rvert = 4$	No
IE:HC - EE:HC	$\lvert 192 - 205\rvert = 13$	No
IE:HC - IE:SD	$\lvert 192 - 392\rvert = 200$	Yes
IE:HC - SE:SD	$\lvert 192 - 438\rvert = 246$	Yes
IE:HC - EE:SD	$\lvert 192 - 231\rvert = 39$	No
SE:HC - EE:HC	$\lvert 196 - 205\rvert = 9$	No
SE:HC - IE:SD	$\lvert 196 - 392\rvert = 196$	Yes
SE:HC - SE:SD	$\lvert 196 - 438\rvert = 242$	Yes
SE:HC - EE:SD	$\lvert 196 - 231\rvert = 35$	No
EE:HC - IE:SD	$\lvert 205 - 392\rvert = 187$	Yes
EE:HC - SE:SD	$\lvert 205 - 438\rvert = 233$	Yes
EE:HC - EE:SD	$\lvert 205 - 231\rvert = 26$	No
IE:SD - SE:SD	$\lvert 392 - 438\rvert = 46$	No
IE:SD - EE:SD	$\lvert 392 - 231\rvert = 161$	Yes
SE:SD - EE:SD	$\lvert 438 - 231\rvert = 207$	Yes

all pairwise comparisons of means, we notice a few things. First, there are statistically significant differences in mean time spent in darkness between those mice who stayed in the home cage and those exposed to stress regardless of whether the mouse was also in an impoverished or standard environment. However, if the mouse was in an enriched environment, there was no statistically significant difference to be found between being exposed to stress and remaining in the home cage.

We did see that, within those mice exposed to stress, those in the enriched environment spent less time in the dark than those in the impoverished and standard environments. These differences were statistically significant. As a whole, we would conclude that putting mice in an enriched environment increases the resiliency of mice in recovering from stress.